QUARTERLY PROGRESS REPORT NO. 3

on

A STUDY OF MAGNETIC AND ACOUSTIC EXCITATIONS AT MICROWAVE FREQUENCIES IN SOLIDS

Contract No. NASW-1184 Period Covered: 1 November 1965 - 31 January 1966

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Prepared for:

NASA Headquarters Office of Advanced Research and Technology, Research Division Code RRE Washington, D.C.

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NASA Headquarters Washington, D.C.

Reference: Contract No. NASW-1184

Subject: Third Quarterly Progress Report,"A Study of Magnetic and Acoustic

Excitations at Microwave Frequencies in Solids", covering the

period November 1, 1965 to January 31, 1966.

Gentlemen:

I. SUMMARY

Our studies of collective wave behavior due to nonlinear effects arising from the strong electron-piezoelectric interaction in amplifying CdS have continued during this quarter. A paper describing the fundamental mechanisms giving rise to the ac collective effects is affixed to this report as an appendix.

Experimental work on magnetostatic and magnetoelastic interactions and a theoretical analysis of the intermediate mode of propagation in YIG rods is also given in this report.

II. MAGNET OACOUSTIC AND SPIN WAVE INTERACTIONS

A. Introduction

Two modes of spin wave propagation have been observed previously in yttrium iron garnet (YIG) rods: the magnetoelastic mode and the magnetostatic mode. However, a new mode of spin wave propagation was observed and reported in the second quarterly progress report written for this contract. This new mode of propagation is of interest not only because it will furnish us with a more complete understanding of magnetic interactions in YIG and similar materials but also from the practical point of view. The linear dispersive characteristics of the new mode are ideal for field or frequency swept delay line devices. During this quarter we have extended our experimental work and refined the model for the interaction by means of which the delay time is calculated.

B. Discussion

1. Experiment

Since it is important to determine where the intermediate echoes are generated, experimental data obtained from rods having polished end faces were compared with that obtained from those with end faces which are unsatisfactory for magnetoelastic studies. It was found that there was no correlation between the perfection of the end surfaces and the appearance of the intermediate echoes. However, if magnetostatic echoes were observed, intermediate echoes were also seen. From these preliminary experiments and the additional fact that both magnetostatic and intermediate echoes are present in a transversely magnetized rod, although magnetoelastic echoes are not seen, we are led to conclude that the magnetostatic and intermediate echoes are generated at similar positions within the rod.

An interesting feature is found in comparing the applied magnetic fields at which the magnetoelastic and the magnetostatic echoes are first seen at a particular temperature. These initial fields (H^i) were compared at various temperatures for three different rods. The results are shown in Table 1. It is seen that the difference $\Delta\mathrm{H}$ between the field at which the magnetoelastic echoes are first observed (H^i_{ME}) and the field at which magnetostatic echoes are observed (H^i_{MS}) is independent of rod length and diameter and is equal to 2π times the saturation magnetization M_S .

$$\Delta H = H_{ME}^{i} - H_{MS} \cong 2\pi M_{S}$$

The most startling piece of information obtained from the data is that the field at which the turning point occurs $(\frac{\omega}{\gamma} = H_{i}^{t})$ is found within rods A and B. If we assume, as is generally done, that magnetostatic waves traverse the whole length of the rod, then magnetostatic echoes should not be observed under these conditions because spin waves cannot propagate in regions where the internal field is greater than the internal field at the turning point $(H_{i} > H_{i}^{t})$. It is important to note that magnetostatic echoes are observed in rods A and B even though it appears that they should not be.

In addition, magnetoelastic echoes should not be observed for applied fields (H_0) greater than

$$H_{ME}^{i} = \frac{\omega}{\gamma} + 2\pi M_{S} .$$

But it is seen in rods A and B that $\begin{array}{ccc} H & \text{is significantly larger than} \\ \cdot & \cdot & \cdot \end{array}$ $H_{MF}^{\hat{1}}$. The clue to this strange behavior is contained in rod C, which does not exhibit these effects, and also in the temperature dependence of H_{ME}^1 in rods A and B. Rods A and C have essentially the same length and diameter and were obtained from the same source (Microwave Chemicals Laboratory) but most likely were grown on different days with slightly different batches of material. Thus we shall assume that the difference in behavior is due to the presence of impurities whose influence manifests itself like an additional demagnetizing field. This assumption is supported by the fact that $H_{ME}^{\dot{1}}$ rises with increasing temperature in samples A and B. This fact indicates that the magnitude of H_{MF}^1 is not a function of the saturation magnetization which decreases with increasing temperature, but is most probably related to some exchange relaxation rate between impurity ions and the iron sublattice. This problem will be studied further, not only to find the source of the effect but also to discover how the anomalous resonance field influences spin wave interactions in YIG rods.

During the course of this work the conditions for maximum amplitude of the intermediate echo were studied. It was found that maximum intensity is obtained when the direction of the r.f. field $\,h_{RE}^{\,\,\dot{}}\,\,$ is perpendicular to the rod axis. Deviations of more than 10 reduce the amplitude considerably. It should be noted that this condition does not apply to the direction of the external field since maximum echo intensity this case can be quite large ($\sim 5 - 10^{0}$) and no consistent behavior was found. Temperature also influences the echo intensity. Although this work is in its initial stages, a maximum in the number and intensity of the intermediate echoes with decreasing temperature was found to occur at

about $170^{0} \rm K$ in rod B. In addition, the intensity of the echoes at $77^{0} \rm K$ was less than that observed at room temperature.

2. Theory

In discussing the mode of propagation of the intermediate echoes we must distinguish clearly between the generating point and the turning point. The fact that this difference exists can be illustrated clearly by discussing briefly the magnetoelastic and magnetostatic modes. The dispersion relation used to describe magnetoelastic waves.

$$\omega = \gamma H_i + \gamma D k^2 \qquad ... 1$$

where ω is the operating frequency, D is the exchange constant and H. is the internal field in the rod, is shown at the top of Fig. 1. The linear dispersion is due to elastic waves ($\omega = ck$) and the parabola is the spin wave exchange branch. Point B at the intersection of the curves is called the crossover region where spin waves and elastic waves interact. We are interested in the manner in which electromagnetic radiation at microwave frequencies is converted into spin waves. The difficulty in coupling to spin waves arises from the fact that the spin wave length is small compared to the r.f. wavelength. However, at the turning point A. which we shall for the time being define as the position in the rod where $\frac{\omega}{M}$ = H_i, several important effects occur. Spin waves cannot propagate beyond $\dot{\mathsf{A}}$ in the direction of the center of the rod (Fig. 1). This is due to the fact that $k^2 = (\frac{\omega}{\gamma} - H_i)/D$ and since $H_i > \frac{\omega}{\gamma}$ the wave number is imaginary $(k^2 < 0)$ and spin waves are damped. In addition, the wavelength of spin waves decreases rapidly due to the decrease in $H_{\hat{\mathbf{i}}}$ as we move from A to B toward the end of the rod as shown in the bottom of Fig. 1. Thus it can be seen that a net spin moment has been provided at the turning point to which the r.f. field can couple. Spin waves which travel toward the turning point are generated in this manner,

MAGNETOELASTIC MODE

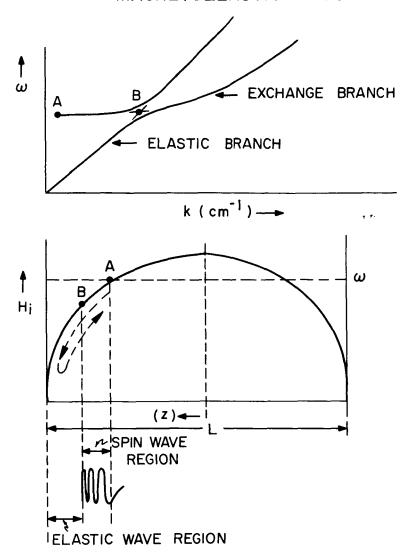


FIG. 1 (Top) Dispersion relation for phonons and exchange-dominated spin waves. The degeneracy is lifted at B, the crossover point where the interaction between phonons and spin waves is strong. Point A at small values of k is called the turning point at which spin waves are reflected and beyond which spin wave propagation does not occur. A net spin moment is also provided at point A due to a wavelength decrease toward point B as shown at the bottom of the figure.

(Center) Internal field configuration in an axially magnetized rod. The turning point A and the crossover point B are shown. The direction of propagation of the magnetostatic mode is shown by the arrows.

In the case of magnetostatic modes the dispersion relation is 3

$$\frac{\omega}{\gamma} = H_i + 2\pi M \left(\frac{Y_i}{R}\right)^2 \frac{1}{k^2} \qquad ... \qquad 2$$

As stated earlier it is assumed that the turning point is no longer in the rod for magnetostatic mode propagation to occur. Solving 2 for \mathbf{k}^2 we obtain

$$k^{2} = \frac{2\pi M \left(\frac{Y_{i}}{R}\right)^{2}}{\left(\frac{\omega}{\gamma} - H_{i}\right)}$$
 ... 3

It is seen from Eq. 3 and the dc internal field distribution within the axially-magnetized rod (center of Fig. 2) that k^2 increases and therefore the spin wavelength decreases from the end of the rod toward the center as shown at the bottom of Fig. 2. In addition, the turning point $(\omega/\gamma=H_1)$ is found at an infinitely large wavenumber $(k\to\infty)$. However, coupling of the microwave r.f. field to magnetostatic spin waves cannot occur at the turning point since no net moment is available there. This is one important difference between the turning point for magnetostatic waves and that for magnetoelastic waves. It is important to note that even in the case of the magnetoelastic mode the turning point does not provide a coupling region. The source of the net moment is the rapid change of wavelength due to the non-uniformity in the internal dc field as pointed out by Schlömann. Thus magnetostatic waves can be generated and detected only at the ends of the rod where the spin moment exists.

In the second quarterly report we proposed a possible explanation for the intermediate echo which consisted of converting slow-moving magnetostatic spin waves into elastic waves at the crossover point. In this model the change in delay time arose from the varying amounts of time spent as a slow-moving magnetostatic wave. We have calculated the value of k in the magnetostatic region degenerate with the crossover $(\omega_{MS}=\omega_{CR})$ and find

MAGNETOSTATIC MODE

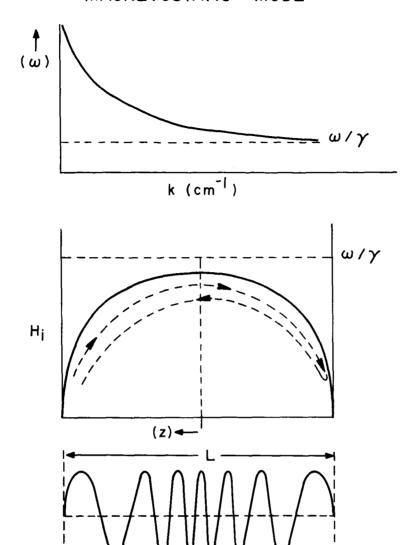


FIG. 2 (Top) Dispersion relation for magnetostatic spin waves showing the asymptotic approach to the turning point at $\,k\,\to\,\infty$.

WAVELENGTH IN ROD

(Center) Internal field distribution in an axially magnetized rod. The direction of propagation of magnetostatic waves is shown by the arrows. The turning point $H_1=\omega/\gamma$ is not contained in the rod.

(Bottom) The relative wavelength of magnetostatic waves in the rod. The large wavelength at the ends of the rod provides the net moment for coupling to the rf field.

$$k = \frac{\sqrt{2\pi MY} \left(\frac{Y_i}{R}\right)}{\sqrt{\frac{D}{h}} k_e} \qquad 4$$

where $\mathbf{k}_{\mathbf{e}}$ is the wavenumber at the crossover point in the exchange region given by

$$k_e = \frac{c \pm \sqrt{c^2 - (\frac{D}{h})^{4}}^{4}}{2\frac{D}{h}} \sim 10^5 (cm^{-1})$$

where c is the velocity of shear elastic waves. Using Eq. 2 the group velocity of magnetostatic waves for the condition $\omega_{MS}=\omega_{CR}$ is found to be

$$\frac{d\omega}{dk} = V_g = -4\pi M \gamma \left(\frac{Y_i}{R}\right)^2 \frac{1}{k^3} = -2.34(10^8) \text{ cm/sec.}$$

Thus we are able to conclude that magnetostatic waves are traveling much too fast in the region degenerate with the crossover to account for the intermediate echo delay time by the above mechanism. However, we have alluded to a magnetostatic turning point as a barrier at which the waves are partially reflected and damped. This can be seen more clearly by writing the equation of motion of magnetostatic waves in the form

$$\frac{\mathrm{d}}{\mathrm{d}z} \left(\frac{\mathrm{d}m}{\mathrm{d}z} \right) + k^2(z)m = 0 \qquad ... 5$$

where k^2 is given by Eq. 3 and z is measured down the axis from the center of the rod. The rate of change of slope (curvature) of Eq. 5 is negative for positive m and $k^2 > 0$, but eventually the m coordinate

INTERMEDIATE MODE

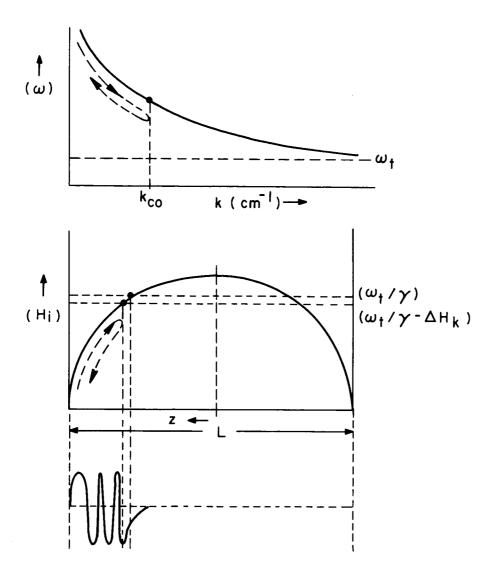


FIG. 3 (Top) Dispersion relation of magnetostatic spin waves showing the cutoff wavenumber arising from losses.

(Center) Internal field distribution in an axially magnetized rod. The turning point without losses (ω/γ) and with losses $(\omega/\gamma-\Delta H_k)$ are shown in addition to the direction of propagation intermediate mode.

(Bottom) Change of wavelength of intermediate mode as we proceed from the end of the rod toward the cutoff wavenumber.

axis will be crossed and m will change sign. This changes the sign of the curvature and the process will repeat, leading to an oscillating function. If, however, we choose the same initial conditions, except that $k^2 \le 0$ and assume a finite value of m at $|z| = \infty$, the function will not cross the m coordinate axis. This is due to the fact that the sign of m and the curvature are the same. As z increases, m approaches zero monotonically (damped function). Thus the character of the wave changes at $\omega = \gamma H_i$. In contrast to the exchange branch where the turning point occurs at k = 0, the magnetostatic turning point occurs at $k = \infty$. This is unsatisfactory for several reasons, not the least of which is the fact that for large values of $\ k$ ($> 10^4~{
m cm}^{-1}$) the dispersion relation is dominated by the exchange energy. We have, however, not included losses in the spin wave dispersion relation. This can be done by replacing $\,\omega\,$ with $\,\omega\,+\,i\gamma(\Delta H_{k}/2)$ and $\,k\,$ with $\,k\,+\,iK\,$ where ΔH_L is the spin wave linewidth.

$$\omega + i \left(\frac{Y \Delta H_k}{2} \right) = Y H_0 + 2 \pi M Y \left(\frac{Y_i}{R} \right)^2 \frac{1}{(k + iK)^2}$$

Equating real and imaginary parts and solving the equation containing the imaginary part for K we obtain

$$K = -k \left[\frac{\gamma_{\Delta} H_{k}}{2(\omega - \gamma H_{i}) + \gamma_{\Delta} H_{k}} \right]$$

substituting K into the equation containing the real part we obtain the new dispersion relation

$$(\omega - \gamma H_{\mathbf{i}}) \left[1 - \frac{(\gamma_{\Delta} H_{\mathbf{k}})^2}{[2(\omega - \gamma H_{\mathbf{i}}) + \gamma_{\Delta} H_{\mathbf{k}}]^2} \right] + \frac{(\gamma_{\Delta} H_{\mathbf{k}})^2}{[2(\omega - \gamma H_{\mathbf{i}}) + \gamma_{\Delta} H_{\mathbf{k}}]} = 2\pi M \gamma \left(\frac{\gamma_{\mathbf{i}}}{R} \right)^2 \frac{1}{\mathbf{k}^2}$$

From Eq. 6 it is seen that the introduction of losses has moved the turning point from $k = \infty$ to a cutoff wavenumber k_{co} given by

$$k_{co} = \sqrt{\frac{2\pi M \gamma \left(\frac{Y_i}{R}\right)^2}{\gamma \Delta H_k}} = 1.8(10^3) \text{ cm}^{-1} \text{ (for } \Delta H_k = 0.1 \text{ Oe)}$$

Magnetostatic wave propagation does not occur for $k > k_{\text{co}}$. However, as we pointed out above the magnetostatic waves are partially reflected at k_{co} in the same way as they are reflected from the turning point in the magnetoelastic mode. The group velocity $V_g = d\omega/dk$ of the intermediate echoes is calculated from Eq. 5 and found to be

$$v_{g} = -\frac{4(\omega - \gamma H_{i})^{3} + 4(\omega - \gamma H_{i})^{2}(\gamma_{\Delta}H_{k}) + 2(\omega - \gamma H_{i})(\gamma_{\Delta}H_{k})^{2} + (\gamma_{\Delta}H_{k})^{3}}{\sqrt{2\pi M \gamma \left(\frac{Y_{i}}{R}\right)^{2} \left\{2(\omega - \gamma H_{i})^{3/2} + 2(\omega - \gamma H_{i})^{1/2}(\gamma_{\Delta}H_{k}) + (\omega - \gamma H_{i})^{-1/2}(\gamma_{\Delta}H_{k})^{2}\right\}}}$$

The delay time τ is obtained from the equation

$$\tau = 2 \int_{\mathbf{z}(\frac{L}{2})}^{\mathbf{z}(\mathbf{k}_{\mathbf{co}})} \frac{d\mathbf{z}}{\mathbf{v}_{\mathbf{g}}}$$

The complexity of the integral has made it difficult to obtain a solution in closed form. However, an order of magnitude estimate has been made by means of a hand calculator. The results of these calculations indicate that our model for the intermediate echo yields satisfactory results.

C. Program for Next Interval

Our analysis of the intermediate echo and the field dependence of the delay time will be completed during the next quarter. A computer program is being written to calculate τ directly. Experimental data will be presented which describes the changes in delay time with temperature and rod size.

In addition, work will continue on the anomalous field effect mentioned in this report. A series of relaxation time measurements will be initiated to gain information about the coupling of impurity ions to the iron sublattice and to determine whether this coupling influences spin wave propagation.

REFERENCES

- 1. W. Strauss, J. Appl. Phys. 35, 1022 (1964); 36, 118 (1965).
- 2. R.W. Damon and H. van de Vaart, Phys. Rev. Letters 12, 583 (1964).
- 3. P.C. Fletcher and C. Kittel, Phys. Rev. <u>120</u>, 2004 (1960).
- 4. E. Schlömann, J. Appl. Phys. 35, 159 (1964).
- 5. W. Strauss, Proc. of the IEEE 53, 1485 (1965).
- 6. M.A. Gintsburg, J. Phys. Chem. Solids 11, 336 (1959).

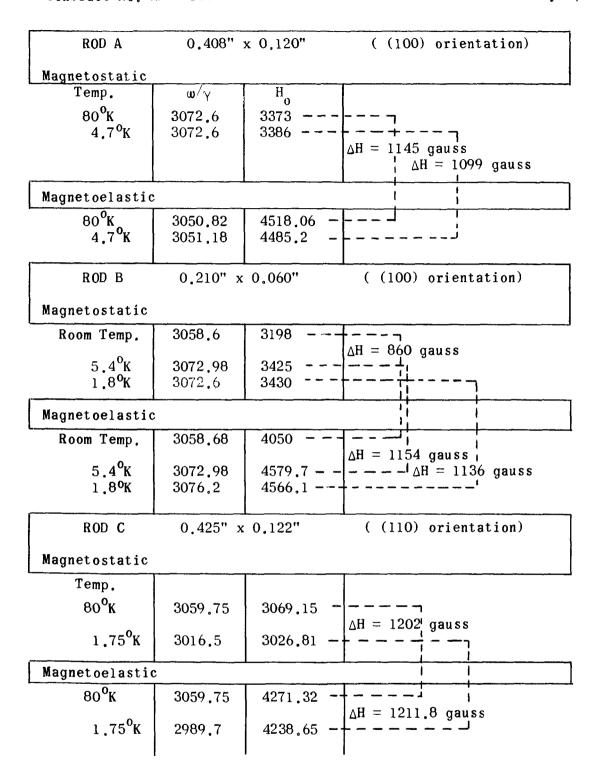


Table I

Magnetic fields at which magnetoelastic and magnetostatic echoes are first observed at various temperatures as the applied field is lowered from values well above the resonance condition.

APPENDIX A

Collective Acoustoelectric Waves in Piezoelectric Materials

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(To be submitted to J. Appl. Phys.)

Collective Acoustoelectric Waves in Piezoelectric Materials

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ABSTRACT

The electron-piezoelectric sound wave interaction gives rise to three distinguishable phenomena: gain, acoustoelectric effects, and nonlinear mixing. The acoustoelectric effects, through coupling to the gain, cause interaction between sound waves which can give rise to collective wave propagation. The propagation can be described as an acoustoelectric wave.

If nonlinear mixing is stronger than the net coherent gain for some group of sound waves, the collective propagation will be a second sound acoustoelectric wave. The nonlinear mixing rate is estimated and found to be rapid enough for second sound waves under certain conditions.

I. INTRODUCTION

The purpose of this paper is to illustrate the fundamental mechanisms which can give rise to ac collective effects in amplifying CdS. The word collective is used to indicate that the effects described are not due to any one sound wave but rather that the effects have to do with the cooperative behavior of many sound waves. Of course, for this cooperative

The work reported here was partially supported by the National Aeronautics and Space Administration under Contract No. NASN-1184.

behavior to come about, some type of interaction must exist between the different sound waves. In CdS, the very strong and extremely nonlinear electronpiezoelectric sound wave interaction provides the means to achieve collective behavior.

In addition to the well-known amplification, the electron-piezoelectric interaction produces two other important phenomena: the acousto-electric effects and nonlinear mixing. The acoustoelectric effects provide the interaction that can hold a collective wave together. The nonlinear mixing can greatly alter the form that the collective wave may take. For very weak nonlinear mixing, the collective wave will be a convective type wave and will travel at close to the velocity of sound. Convective waves are taken to be waves which can grow im amplitude down the length of a crystal. In the limit of large nonlinear mixing, the wave will be a compression wave or second sound wave and will travel at $1/\sqrt{3}$ times the velocity of sound. It appears that both waves have been observed in CdS.

In Sec. II, the electron-piezoelectric wave is shown to give rise to currents which are responsible for: acoustoelectric effects, gain, and nonlinear mixing. In Sec. III, it is shown that under certain conditions acoustoelectric effects can give rise to collective wave propagation.

The effect of the nonlinear mixing is discussed in Sec. IV., and the concept of a sonic fluid is described. The sonic fluid is a valid description of the hypersonic system in the limit of rapid mixing collisions.

The collective wave (or second sound wave), which is the proper solution in the limit of rapid mixing, is described in Sec. V.

In Sec. VI, a rough order of magnitude is calculated for the lifetime of sound waves making up the sonic fluid. On the basis of this lifetime, one can determine which sound wave frequencies make up the sonic fluid and hence are capable of supporting second sound waves.

II. ELECTRON-PIEZOELECTRIC WAVE INTERACTION

The passage of a sound wave through the piezoelectric crystal causes a periodic variation of the electric potential. This variation in potential bunches free carriers in the region of lowest potential. For a crystal which has many sound waves present with frequencies ω_{ℓ} , the electron density distribution will be modulated by each wave ℓ , and one could write the time dependence of the electron density as

$$n = n_0 + \sum_{\ell} n_{\ell} e^{i\omega_{\ell}t} + c.c. \qquad (2.1)$$

where n_0 is the ambient density and n_{ℓ} is the size of the fluctuation associated with each wave ℓ .

Similarly, the carrier drift velocity is modulated by the alternating piezoelectric fields and the drift velocity can be written

$$v_{d} = v_{o} + \sum_{\ell} v_{\ell} e^{i\omega_{\ell} t} + c.c.$$
 (2.2)

where v_0 is the dc current and v_{i} is the size of the fluctuation in velocity due to wave i.

The current is proportional to the product of n and \mathbf{v}_{d} .

$$j = e n_{0} v_{0} + \left(e n_{0} \sum_{\ell} v_{\ell} e^{i\omega_{\ell} t} + e v_{0} \sum_{\ell} n_{\ell} e^{i\omega_{\ell} t} ... \right)$$

$$+ \left[\sum_{\ell} e v_{\ell} n_{\ell} e^{i\omega_{\ell} t} e^{-i\omega_{\ell} t} ... \right]$$

$$+ \left[\sum_{nm} e n_{n} v_{m} e^{i(\omega_{n} \pm \omega_{m})t} ... \right]$$

$$(2.3)$$

$$= j_0 + j(\omega_L) + j'(0) + j'(\omega_L) . \qquad (2.4)$$

The power generated at a frequency ω_{ℓ} is $P(\omega_{\ell}) = \frac{1}{2} Re[j(\omega_{\ell}) E^*(\omega_{\ell}) + c.c.]$ where $E(\omega_{\ell})$ is the piezoelectric field of sound wave ℓ . Whether the wave is being amplified or damped depends on the relative phase between j and E.

The first order currents $j(\omega_{\underline{\ell}})$ are responsible for the familiar ultrasonic amplification of wave ℓ when $v_0 > v_s$. The third term in Eq. (2.4) is the generalized acoustoelectric current. One can see that

$$j'(0) = e\sum_{\mathbf{L}} v_{\mathbf{L}} n_{\mathbf{L}}$$
 (2.5)

is additive over all sound waves $\boldsymbol{\ell}$. One can also see that since j'(0) does not have time dependence $\boldsymbol{\omega}_{\boldsymbol{\ell}}$ it does not, to first order, directly alter $P(\boldsymbol{\omega}_{\boldsymbol{\ell}})$.

The currents $j'(\omega_{\underline{\ell}})$ are the nonlinear mixing currents. For example, if $\omega_{\underline{m}} = \omega_{\underline{n}}$ in Eq. (2.3), then power can be fed to a wave $\omega_{\underline{\ell}}$ where $\omega_{\underline{\ell}} = 2\omega_{\underline{n}}$, giving rise to second harmonic generation. In general, if there is a large number of waves of many frequencies $\omega_{\underline{m}}$, then there will be current at all sum and difference combinations. This can cause mixing of energy to occur among the distribution of sound waves.

To summarize this section, we note that there are, in the presence of piezoelectric sound waves: (1) first order currents which can cause gain or loss, (2) an acoustoelectric current which is proportional to the total sonic energy density in a region, and (3) mixing currents at the sum and difference frequencies for all the waves present.

III. THE ACOUSTOELECTRIC COLLECTIVE WAVE

In this section, we will assume that the effect of the mixing currents is small and we will concentrate on the effect of j'(0), the acousto-electric current. When the electrons are amplifying sound waves, the reaction back on them tends to decrease the current. Therefore, as pointed out by Hutson, the acoustoelectric current then flows in the opposite direction to that of the sound wave. Since the local acoustoelectric current is proportional to the local acoustic energy density, a variation of acoustic density from region to region of the crystal will cause this current to also vary from region to region.

The situation is illustrated in Fig. 1 where a fluctuation or amplitude modulation is set up over distances large compared to the wavelengths of the individual ultrasonic waves. To maintain current continuity, additional electric-field-driven current must flow in the forward direction to cancel out the variations in backward-flowing acoustoelectric current. Current uniformity is achieved by bunching charge in the regions of maximum acoustic energy gradients. The electrostatic fields of the bunched charge add to the local field so as to maintain current continuity.

Using Poisson's equation and the continuity equation, it has been shown by $Carleton^3$ that the local electric field is:

$$E(x) = E_0 + \frac{1}{\sigma} \frac{J_0^{ae} - j^{ae}(x)}{1 - i(\omega/\omega_e)}$$
 (3.1)

where E_0 is the external applied field, σ the conductivity, J_0^{ae} the uniform acoustoelectric current, and $j^{ae}(x)$ the local current which is caused by local fluctuations in sonic intensity. This electric field fluctuation alters the local gain of a sound wave because the gain is a function of μE . The

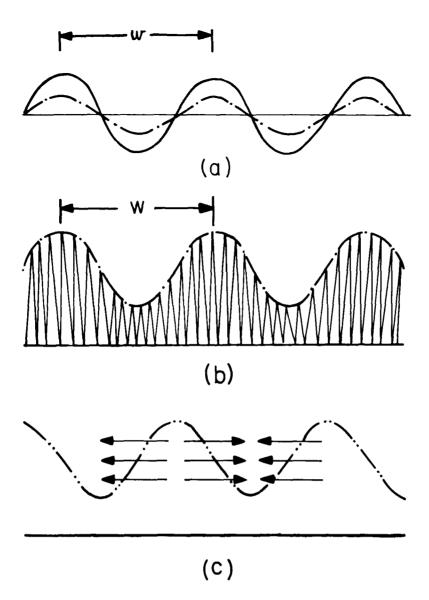


FIG. 1 (a) The large-amplitude sine wave (solid line) with wavelength we represent the electric potential associated with a coherent sound wave in piezoelectric material. The accumulation of electrons in regions of positive potential reduces the potential. The sound wave, with accumulated charge, is represented by the broken-line sine wave. The largest amount of charge accumulation which can occur would completely counterbalance the piezoelectric potential, in which case no further electron attraction occurs.

(b) The broken line represents the envelope of a fluctuation in amplitude and density of pizeoelectric waves. There is no coherent electric potential associated with this envelope. The local electric potentials are associated with the short-wavelength sound $\omega\!<\!\!<\!\!W$ waves and are represented by the jagged lines contained within the density fluctuation. The individual electrons interact with the potentials of individual sound waves giving rise to a varying acoustoelectric effect. This causes a bunching charge which is necessary to maintain current continuity.

(c) The charge accumulation caused by the sonic wave diagrammed in (b) is the doubly broken line. The arrows represent the fields due to the charge bunching. The field is largest in the region of larger sonic energy density. This in turn causes the gain to be greatest in those regions which amplify the initial distribution of sonic energy.

simplest case to discuss is that for which slight increases in the external accelerating field increase the gain linearly (i.e., $\mu E_0 \approx v_s$). The gain is then greatest in a region where the fluctuation in the acoustoelectric field adds to the dc field.

Consider the result of this acoustoelectric effect on an individual sound wave entering a region of somewhat higher acoustic energy density. The sound wave experiences an increase in gain over that which would have occurred had the density fluctuation not been present. The presence of a positive increment in acoustic density causes a positive increment in gain. Since more sonic energy is generated where larger energy densities already exist, the overall effect is to give rise to an apparent attractive force between acoustic elements. This acoustoelectric interaction is established in times of the order of an electron collision time and can therefore be taken to be an instantaneous and local interaction for frequencies and wavelengths comparable to ultrasonic sound waves.

The Boltzmann equation for the energy density \boldsymbol{S}_{k} for a given sonic wave with wavevector \boldsymbol{k} can be written as

$$\frac{\partial}{\partial t} S_k + C_k \cdot \nabla S_k = \alpha_k S_k \tag{3.2}$$

where C_k is the velocity and α_k is the gain of wave k. We shall assume that k is restricted to such values that the classical analysis of White, 1 or Spector, 4 are appropriate (i.e., k $\Lambda \ll 1$, where Λ is the electron mean free path). The interaction of these sound waves with electrons can then be treated as a very smooth interaction over many electron free paths.

We now assume the existence of some zero order distribution of acoustic energy density which is stationary in time but may show convective

growth; that is, under acoustic gain, the anode end of the crystal will have a larger acoustic energy density than the cathode. In this approximation, we may write

$$S_k(x,t) = S_k^0(x) + S_k'(x,t)$$
 (3.3)

The zero order solution for Eq. (3.2) is a difficult nonlinear problem. For a discussion, see Carleton et al. 3 We will assume that all the fluctuations in time are small, i.e., $S_k^\prime \ll S_k^0$ and that Eq. (3.2) may be linearized in S_k^\prime to give

$$\frac{\partial}{\partial t} S_k' + C_k \cdot \nabla S_k' = \alpha_k' S_k^0 + \alpha_k^0 S_k'$$
 (3.4)

where $\alpha_{k}^{'}$ is the alteration in gain which may accompany any variation in $S_{k}^{'}$.

The variation in α_k as a function of electron drift velocity for low frequency sound waves has been derived by White and Spector. To simplify our problem, we will assume that E_0 is such that a linear dependence of α on v_1 is a good approximation for small enough v_1 , where v_1 is the variation in the drift velocity v_d . This is the case for $v_d \approx C_k$. We will also assume, within the classical context of this discussion, that all nonlinearities between v_d and E_0 , the externally applied accelerating field, are due to a build-up of acoustoelectric field E^{ae} , i.e., μ is constant. In this approximation, one can write

$$\alpha'_{\mathbf{k}} = \alpha'' \mathbf{E}_{\mathbf{ae}}$$
 (3.5)

The acoustoelectric current can be written in terms of α and S and is

$$j_{ae} = -\frac{\sigma}{enc} \sum_{k} \alpha_{k} S_{k}$$

$$= -\frac{\sigma}{enc} \sum_{k} (\alpha_{k}^{0} S_{k}^{0} + \alpha_{k}^{\prime} S_{k}^{0} + \alpha_{k}^{0} S_{k}^{\prime} \dots)$$
(3.6)

where n is the dc electron density and c is the velocity of sound.

The resulting local field variation is

$$\mathbf{E}_{\mathbf{ae}}' = \frac{1}{\mathbf{enc}} \sum_{\mathbf{k}} (\alpha_{\mathbf{k}}^{0} \mathbf{S}_{\mathbf{k}}' + \alpha_{\mathbf{k}}'' \mathbf{E}_{\mathbf{ae}}' \mathbf{S}_{\mathbf{k}}^{0})$$
 (3.7)

$$\mathbf{E}_{ae}' = \frac{\Sigma_{k} \alpha_{k}^{0} S_{k}'}{\operatorname{enc} - \Sigma_{k} \alpha_{k}'' S_{k}^{0}}$$
(3.8)

Equation (3.4) then becomes

$$(\frac{\partial}{\partial t} + C_{\mathbf{q}} \cdot \nabla - \alpha_{\mathbf{q}}^{\mathbf{0}}) S_{\mathbf{q}}' = \alpha_{\mathbf{q}}'' S_{\mathbf{q}}^{\mathbf{0}} E_{\mathbf{a}\mathbf{e}}'$$

$$= \frac{\alpha_{\mathbf{q}}'' S_{\mathbf{q}}^{\mathbf{0}}}{\operatorname{enc} - \sum_{\mathbf{k}} \alpha_{\mathbf{k}}'' S_{\mathbf{k}}^{\mathbf{0}}} \sum_{\mathbf{k}} \alpha_{\mathbf{k}}^{\mathbf{0}} S_{\mathbf{k}}'$$
(3.9)

One can derive an equation for a collective coordinate $\sum\limits_k \alpha_k^0 S_k'$ by multiplying Eq. (3.9) by α_k^0 and summing. We assume plane wave solutions for the density fluctuation, i.e.,

$$S_{q}' = S_{q}'' e^{i(Qx-\Omega t)} e^{\beta x}$$
 (3.10)

The dispersion relation for the collective coordinate then becomes

$$(enc - \sum_{k} \alpha_{k}^{"} S_{k}^{0}) = \sum_{k} \frac{\alpha_{k}^{0} \alpha_{k}^{"} S_{k}^{0}}{i (C_{k}^{0} - \Omega) + (\beta' - \alpha_{k}^{0})}$$
(3.11)

where $\beta' = C_k \cdot \beta$ and k and q, both dummy indices, have been replaced by k. Upon separating real and imaginary parts

$$\sum_{k} \alpha_{k}'' S_{k}^{0} \frac{\alpha_{k}^{0} (C_{k} \cdot Q - \Omega)}{(C_{k} \cdot Q - \Omega)^{2} + (\beta' - \alpha_{k}^{0})^{2}} = 0$$
 (3.12)

and

$$-\text{enc} + \sum_{k} \alpha_{k}^{"} S_{k}^{0} \qquad \left(1 + \frac{\alpha_{k}^{0} (\beta' - \alpha_{k}^{0})}{(C_{k} \cdot Q - \Omega)^{2} + (\beta' - \alpha_{k}^{0})^{2}}\right) = 0 \qquad (3.13)$$

In the limit of very large gain by individual sound waves, i.e.,

$$\left| (\beta' - \alpha_k^0) \right| \gg (C_k Q - \Omega), \qquad (3.14)$$

equation (3.12) gives the linear dispersion relation

$$\Omega = \frac{\left(\sum_{k}^{\infty} \frac{\alpha_{k}^{0} \alpha_{k}^{''} S_{k}^{0} C_{k} \cos \theta_{k}}{(\beta' - \alpha_{k}^{0})^{3}}\right)}{\left(\frac{\alpha_{k}^{0} \alpha_{k}^{''} S_{k}^{0}}{(\beta' - \alpha_{k}^{0})^{3}}\right)} \qquad Q$$

$$(3.15)$$

where θ_k is the direction sound wave k makes with the collective wavevector Q. The collective wave velocity is some weighted average of the single sound

wave velocities, C_k . In the same limit, Eq. (3.13) gives

$$\sum_{k} \alpha_{k}^{"} S_{k}^{0} \left(\frac{\alpha_{k}^{0}}{\beta' - \alpha_{k}^{0}} \right) = \text{enc} - \sum_{k} \alpha_{k}^{"} S_{k}^{0}$$

$$(3.16)$$

The term "enc" is approximately the dc current, whereas $\sum\limits_k \alpha_k'' S_k^0$ is the local current, which is the same size as the acoustoelectric current. We therefore assume

$$enc > \sum_{k} \alpha_{k}^{"} S_{k}^{0}$$
 (3.17)

which implies

$$\beta' > \alpha_k^0 \tag{3.18}$$

Equation (3.15) then becomes

$$\Omega \approx \frac{\sum_{\mathbf{k}} \alpha_{\mathbf{k}}^{\mathbf{0}} \alpha_{\mathbf{k}}'' S_{\mathbf{k}}^{\mathbf{0}} C_{\mathbf{k}} \cos \theta_{\mathbf{k}}}{\sum_{\mathbf{k}} \alpha_{\mathbf{k}}^{\mathbf{0}} \alpha_{\mathbf{k}}'' S_{\mathbf{k}}^{\mathbf{0}}} \quad Q \approx \overline{C}Q$$
(3.19)

and Eq. (3.16) becomes

$$\beta' = \frac{\sum\limits_{k} \alpha'_{k} \alpha_{k}^{0} S_{k}^{0}}{\operatorname{enc} - \sum\limits_{k} \alpha''_{k} S_{k}^{0}}$$
(3.20)

In CdS the ratio of $\frac{\alpha_k}{\lambda_k}$, where λ_k is the wavelength $\frac{2\pi}{k}$, is of the order of one per cent for ultrasonic frequencies. The approximation (3.14) would then be valid for collective wave frequencies

$$\Omega < 10^{-2} \omega_{\mathbf{k}} \tag{3.21}$$

where $\boldsymbol{\omega}_{\boldsymbol{k}}$ is the frequency of the participating ultrasonic waves.

A rough approximation to the dispersion relation when condition (3.14) does not hold can be found by an approximation of the product $\alpha_k'' \ S_k^0 \ \alpha_k^0 \ .$

The dc gain α_k^0 is positive only within a cone such that the component of electron drift velocity along a sound wavevector is greater than C_k . Similarly, one expects the dc sonic energy density S_k^0 to be large where $\alpha_k^0 > 0$ and very small for $\alpha_k^0 < 0$. We then expect the major contribution to the product α_k'' S_k^0 α_k^0 to occur within the gain cone and only a small contribution from outside. We approximate this by setting

$$\alpha_{k}^{"} S_{k}^{0} \alpha_{k}^{0} = \begin{cases} A & \text{for } \theta_{k} < 3 \\ 0 & \text{for } \theta_{k} > 3 \end{cases}$$

$$(3.22)$$

Changing the sum in Eq. (2.12) to an integral and concentrating only on the angular integration, one gets

$$\frac{2\pi V}{8\pi^3} \int_{\mu^0}^{1} \frac{(C Q\mu - \Omega) d\mu}{(C Q\mu - \Omega)^2 + \mu^2 (C\beta - \alpha^{00})^2} = 0$$
 (3.23)

where $\mu=\cos\theta_k$, $\mu^0=\cos\theta$, and $\mu\alpha^{00}=\langle\alpha_p^0\rangle$, where $\langle\ \rangle$ signifies some average value. Equation (3.23) becomes

$$\log \left[\frac{(CQ - \Omega)^{2} + (C\beta - \alpha^{00})^{2}}{(CQ \mu^{0} - \Omega)^{2} + \mu^{02} (C\beta - \alpha^{00})^{2}} \right]$$

$$= \frac{\alpha^{00} - C\beta}{CQ} tan^{-1} \left[\frac{4\Omega(1 - \mu) (\alpha^{00} - C\beta)}{4\Omega(\Omega - \frac{1+\mu}{2} CQ) + \mu^{2} [C^{2}Q^{2} + (\alpha^{00} - C\beta)^{2}]} \right] (3.24)$$

In the limit C $_{\rm B}$ - α^{00} << CQ, the right-hand side of Eq. (3.24) is of the order

$$\left[\frac{C\beta - \alpha^{00}}{CQ}\right]^2 << 1 .$$

The left-hand side then gives

$$\Omega \approx \frac{1 + \mu^{0}}{2} CQ + \frac{1 + \mu^{0}}{2} \frac{C\beta - \alpha^{00}}{CQ} (C\beta - \alpha^{00})$$

$$\approx \overline{C}Q$$
(3.25)

which is roughly the same averaged velocity to first order as Eq. (3.19).

A similar approximation to the integral can be made in Eq. (3.13) and, with
the substitution of Eq. (3.25), it becomes

$$\tan^{-1} \left[\frac{4\Omega (1 - \mu^{0}) (C \cdot \beta - \alpha^{00})}{4\Omega (\Omega - \frac{1 + \mu^{0}}{2} CQ) + \mu^{0} [C^{2}Q^{3} + (C \cdot \beta - \alpha^{00})^{2}]} \right]$$

$$= \frac{2 C \cdot Q}{A \left(1 + \frac{C^{2}Q^{3}}{C^{2}Q^{2} + (C\beta - \alpha^{00})^{2}}\right)} \left\{ \sum_{k} \alpha''_{k} S_{k}^{0} - enc \right\}$$
(3.26)

or

$$\beta = \frac{\alpha^{00}}{C} - \frac{1}{2} \left(\frac{\mu^{0}}{1 - \mu^{0}} \right) \frac{CQ^{2}}{A} \left\{ enc - \sum_{k} \alpha_{k}'' S_{k}^{0} \right\}$$
 (3.27)

IV. THE EFFECT OF NONLINEAR MIXING

In the previous section, we have ignored the existence of the non-linear mixing currents. This approximation becomes particularly poor as the frequencies of the sound waves increase. To help in understanding the role of these nonlinear currents, it is best to think of the sound wave spectrum as being made of three regions as shown in Fig. 2.

In the lowest frequency range, with wavevector k such that $0 < k < k_1$, are the sonic and ultrasonic frequencies which have large coherent gain under amplifying conditions. Although these low frequency sound waves are coupled to other sound waves so as to generate harmonics, etc., their amplitude has net growth with time. At the other extreme, the highest frequency range, consisting of "thermal phonons" with $k > k_2$, is only slightly affected by the crystal being in gain condition. The reason for this is that sound waves at these high frequencies are dominated by nonlinear interactions (piezoelectric and other) which couple these modes to the crystal thermal reservoir. Pumping energy into these sound modes merely causes a slight increase in the ambient temperature of the crystal.

The intermediate frequencies $\mathbf{k_1} < \mathbf{k} < \mathbf{k_2}$, which for convenience we will call the hypersonic region, are those in which nonlinear mixing and coherent gain compete and one cannot ignore the role of mixing terms. The formula for nonlinear mixing between piezoelectric waves has been derived by several authors. The mixing is found to be much stronger than the nonlinearities associated with crystal anharmonicities. In the hypersonic frequency range, the dominant nonlinear term is that of the nonlinear currents. This couples piezoelectric hypersonic waves among themselves more strongly than to other modes of the system. The coupling to piezoelectric sound waves of thermal frequencies is not strong because the bunching of charge required for these short wavelength currents is opposed by diffusion effects.

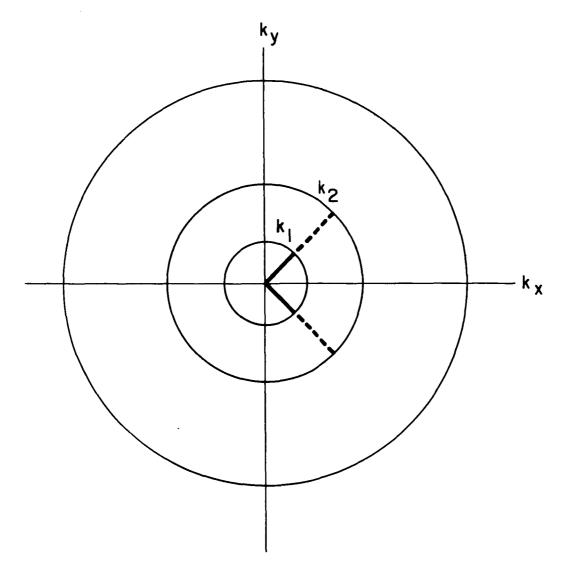


FIG. 2 A convenient breakdown of the total piezoelectric sound wave spectrum into ultrasonic, hypersonic, and thermal regions. The ultrasonic waves with wavevector $k < k_1$ have large coherent gain. The hypersonic region $k_1 < k < k_2$ has losses comparable to the coherent gain. The thermal region $k > k_2$ has net loss. The angular cone about the direction of electron drift $k_{\boldsymbol{x}}$ is the limit of the gain cone.

The nonlinear interaction tends to bring any given sound mode into equilibrium with those coupled to it. By contrast, the coherent amplification tends to increase the energy of a mode and to take it further out of equilibrium. For our purposes, we shall define the hypersonic region $(k_1 < k < k_2)$ as that region in which the nonlinear mixing is just dominant over the coherent net gain. Such sound waves are then damped to a level at which they are equilibrated with the other hypersonic modes. However, the sum of all such modes is undergoing net gain from the drifting electrons. Such a band of sound waves can no longer be treated by Eq. (3.2) where each mode k is well defined and identifiable over times of the period of the collective wave. When the mixing is strong enough, the modes exchange energy rapidly and one can best describe the system as a fluid.

Those modes in the gain cone of Fig. 2 (where $v_c \cos\theta > v_s$) are amplified. In the hypersonic region, energy is pumped into those modes in the gain cone, but is rapidly distributed over all the modes which are coupled via the nonlinear interaction. If the net gain exceeds the loss for the entire shell, then one can describe these modes as a fluid which is gaining in total energy and net sonic momentum.

V. THE SECOND SOUND WAVE

Collective waves do exist in the fast mixing limit; i.e., the time, τ_m , needed for a single particle to lose e^{-1} of its energy to other particles is short compared to periods of interest $(\Omega\tau_m<1)$. The most familiar example is that of sound waves in particle gases. One does not use directly the equations of motion of individual molecules, analogous to Eq. (3.2), but rather one uses hydrodynamic variables such as pressure, drift velocity, density, etc. The propagation of a compressional wave like a sound wave in

a particle gas has been examined for a fluid of hypersonic phonons. The analogy with a sound wave holds very well giving rise to the name "second sound" for the phenomenon.

When the density of sonic energy is higher in one region than in another, the sonic radiation pressure is also higher. Sonic energy will then tend to flow down the radiation pressure gradient. The velocity of a compressional wave in such a situation would be given by

$$A_s = \frac{9b}{9b}$$

where p is the radiation pressure and ρ is the effective mass of the sound waves. If the particles are sound waves, they all have the same velocity c and

$$p = \rho c^2 \cos^2 \theta = \frac{1}{3} \rho c^2$$

where $\cos\theta$ is the angle a hypersonic wave makes with normal to the surface over which the pressure is taken. The average of $\cos^2\theta$ is $\frac{1}{3}$ when the hypersonic waves are uniformly distributed over all angles. This distribution results from the mixing interactions which tend to return the hypersonic fluid to a thermal distribution. Therefore

$$v = \frac{1}{\sqrt{3}} c \cdot$$

The hydrodynamic, or rather thermodynamic, wave described here is different from the dynamic waves described in Sec. 3. However, the acousto-electric effect will still be present. Fluctuations in hypersonic density will cause variations in the acoustoelectric field. The major observable difference between the two limits of collective waves is in the velocity of propagation. For a more thorough and rigorous description of this type

of collective wave the reader is referred to the literature on second sound in solids. 7

VI. THE MEAN FREE PATH FOR MIXING COLLISIONS

Second harmonic generation, a special case of the nonlinear interaction, has been investigated by several authors. 5,6 The following equation has been derived by Kroger

$$\frac{\partial p_{a\omega}}{\partial x} = \frac{K^{a}\omega_{c}\mu}{2\varepsilon c_{o}^{a}} \frac{\left(Y + i\frac{\omega}{\omega_{D}}\right)p_{\omega}^{a}}{\left[Y + i\left(\frac{\omega_{c}}{\omega} + \frac{\omega}{\omega_{D}}\right)\right]^{a}\left[Y + i\left(\frac{\omega_{c}}{2\omega} + \frac{2\omega}{\omega_{D}}\right)\right]}$$

$$= A_{a}(\omega)p_{\omega}^{a}$$
(6.1)

where $\boldsymbol{p}_{\boldsymbol{z}\omega}$ is proportional to the amplitude of the second harmonic,

$$K^{a} = \frac{e_{D}^{a}}{c\varepsilon_{O}} \quad \omega_{C} = \frac{\sigma}{\varepsilon_{O}} \quad \omega_{D} = \frac{c^{a}}{D_{n}}$$

$$Y = 1 - \frac{\mu E_{O}}{c}$$
(6.2)

where e $_p$ is the piezoelectric constant, ε_o the dielectric constant, σ is the conductivity, μ the mobility, D_n the electron diffusion constant, and p_ω is proportional to the amplitude of the initial wave.

To simplify greatly the following analysis, we will suppress the frequency dependence of the interaction by assuming $\omega \approx \omega_{_{\scriptstyle C}} \approx \omega_{_{\scriptstyle O}}$. This assumption is not unreasonable and can be satisfied in photoconducting CdS for $\omega \approx 10^{^{10}}$ cps.

This equation can then easily be generalized to the nonlinear interaction of three sound waves as long as all three frequencies are roughly equal. Equation (6.1) becomes

$$\frac{\partial}{\partial t} p_{\omega_{1}} = \frac{K^{2} \omega_{c} \mu}{2\varepsilon_{0} c_{0}^{2}} p_{\omega_{2}} p_{\omega_{3}}$$
 (6.3)

where $\omega_2 + \omega_3 = \omega_1$. The waves ω_2 and ω_3 are being damped by transferring energy to ω_1 . The mean free path for energy transfer between three sound modes, assuming p's given by thermal amplitude at room temperature for reasonable values in photoconducting CdS, is

$$\ell \approx 10^7 \text{ cm}$$
.

The actual mean free path is given approximately by summing the interaction with a given pair of waves over all the pairs capable of interacting.

The importance of considering the hypersonic region separately arises because of the possibility of a bootstrap mechanism determining the lifetime or mean free path of these sound waves. The piezoelectric-electronic nonlinear interaction is much stronger than the usual crystal anharmonic interactions. This makes it possible for this interaction to be the dominant mechanism for determining the lifetime of the hypersonic waves. Because this mechanism is dominant, we can restrict the modes with which we are concerned to only those piezoactive modes up to wavevector \mathbf{k}_2 which are strongly coupled by the nonlinear currents.

Sound waves with long mean free paths and those being amplified tend to interact only with collinear modes. As shown by Eq. (6.1), the flow of energy in such an interaction depends on the relative phase of the

participating waves. When the waves are not collinear, the relative phase alternates because the wave fronts do not keep pace. This causes the net flow of energy to cancel, giving rise to frequency and wavevector conservation laws. Sound waves with short mean free paths and short lifetimes, however, can interact with a wider group of waves as complete cancellation does not occur during the short times of the interaction, as pointed out by Nava et al. and Ciccarello and Dransfeld. The situation then is:

(a) The lifetime of the waves depends strongly on the number of hypersonic waves which can cause damping via the piezoelectric anharmonic terms, and (b) The number of modes capable of interaction by the piezoelectric anharmonicity depends strongly on the lifetime. The angle that wavevectors can make with one another and still interact due to finite mean free paths is illustrated in Fig. 3.

To avoid cancellation of energy transfer, the wavevectors must add up to a closed triangle; otherwise, a beat vector is introduced. However, for finite mean free paths, the wavevectors can miss adding to zero provided the resultant vector Δ is such that

$$\Delta L < 1$$

where L is the mean free path over which the interaction takes place. Assuming that Δ is specified by having components along vectors b and c in Fig. 4.

$$\cos\theta \approx \frac{\left(c + \Delta_{c}\right)^{2} - \left(b + \Delta_{b}\right)^{2} - a^{2}}{2\left|a\right| \left|b + \Delta_{b}\right|}, \qquad (6.4)$$

and assuming approximate conservation of energy

$$a^{3} + b^{3} = c^{3}$$
 (6.5)

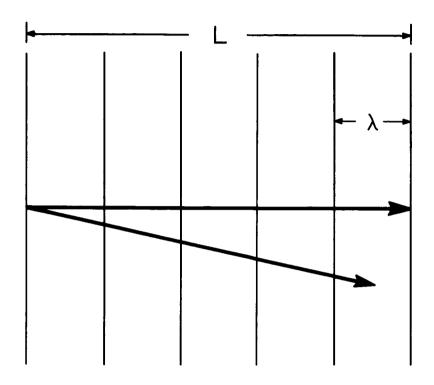


FIG. 3 One of the arrows represents a piezoelectric sound wave with wavelength λ . The other arrow represents a nonlinear mixing current generated by two other sound waves and has roughly the same frequency as the initial wave. The distance L is the mean free path for all sound waves in question. It can be seen that the current and field after both wave and current have traveled a distance L do not have their phase fronts dephased by more than $\lambda/2$, and that the direction of the flow of energy will not necessarily have cancelled out due to dephasing.

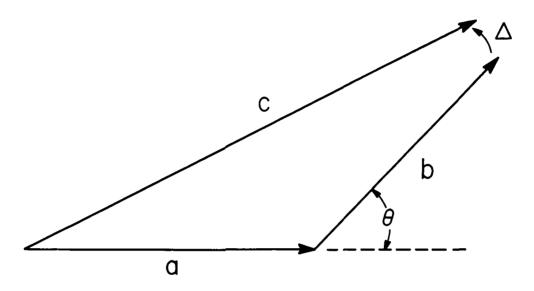


FIG. 4 A possible way for energy to be transferred from a sound wave with wavevector a to sound waves b and c. The difference of b-c need not equal a exactly as some uncertainty in the conservation of wavevector arises due to finite mean free paths (see Fig. 3). If Δ , the uncertainty, is chosen as large as possible and is oriented properly, an angle θ is defined such that θ is the largest possible departure of b from being colinear with a Then, every wavevector contained in the cone generated by θ can contribute to a three sound wave interaction which can contribute to the damping of wave a .

and that $\Delta_{\rm b}~\approx~\Delta_{\rm c}$ and a \sim b \sim c .

$$\cos \theta_{\text{max}} \approx \frac{1 - \frac{\Delta}{a}}{1 + \frac{\Delta}{a}} \approx \frac{1 - \frac{\lambda}{L}}{1 + \frac{\lambda}{L}}$$
 (6.6)

where λ is the wavelengths involved.

The angle θ_{max} defines a cone about a given wave a. Every sonic mode with wavevector within this cone represents a three wave interaction possible for sonic wave a. All can contribute to damping of wave "a".

An estimate of the mean free path L can be determined selfconsistently using

$$\frac{dx}{L} = \frac{dx}{\ell} N(\theta_{max}) = \frac{dx}{\ell} N \frac{\Omega}{4\pi}$$
 (6.7)

where $\frac{dx}{L}$ is the loss of energy from a wave during the traversal of the infinitesimal distance dx, $\frac{dx}{2}$ is that lost in a given three wave interaction, $N(\theta_{max})$ is the number of waves in the cone θ_{max} , and Ω is the solid angle subtending θ_{max} and

$$\Omega = 1 - \cos\theta_{\text{max}} \quad . \tag{6.8}$$

Combining Eqs. (6.6) - (6.8), we find

$$L = \frac{\ell}{N} \frac{1}{(2\pi)^{-1} - \frac{\ell}{N\lambda}} \qquad (6.9)$$

The total number of modes N should be restricted to those modes which are very strongly coupled. This would be the piezoelectrically active

modes with wavevectors up to \mathbf{k}_2 in Fig. 2. Carrier diffusion reduces the ability to bunch carriers at shorter wavelengths, reducing the nonlinear current. Therefore,

$$N \approx \frac{k_2^3}{2\pi} \approx \frac{4\pi}{\lambda^3} \tag{6.10}$$

where λ is roughly the same wavelength as that with which we have been dealing all along.

$$L \approx \frac{\ell \lambda^3}{4\pi} \frac{4\pi}{2 - \ell \lambda^2} \tag{6.11}$$

Assuming that the upper frequency corresponding to ${\bf k_2}$ in CdS is $\upsilon>10^9$, the term $\text{L}\lambda^2$ < 2 and

$$\frac{L}{\lambda} \approx 4\lambda^2 \approx 1$$
 (6.12)

This mean free path is so short that the approximation made obviously no longer holds. However, the calculation does indicate that mean free paths can be greatly shortened by the piezoelectric nonlinear interaction for hypersonic waves with little or no net coherent gain. For lower frequencies, Eq. (6.11) appears to become infinite or negative. This is due to the use of a continuous formula for Ω in Eq. (6.6) which allows for apparent fractional mode interaction. This is a nonphysical result because at least a number of collinear harmonic interactions can occur and

$$L \lesssim L$$
 (6.13)

For very short mean free paths, $\frac{L}{\lambda}\approx 1$, the proper description of the system is that of a well-mixed gas and the normal modes are the second sound collective modes.

VII. CONCLUSION

The strong electron-piezoelectric interaction, which makes acoustic amplification possible in materials such as CdS, also produces non-linear interactions between sound waves which can cause collective phenomena.

The acoustoelectric effect is an always present form of interaction which causes gain to be highest in regions of highest acoustic density. Any initial fluctuation in acoustic energy will be enhanced by this variable gain. In particular, a sinusoidal distribution of acoustic energy in space will be amplified as it travels down the crystal in the direction of the current. This wave may be composed of many individual sound frequencies and does not, except under unusual circumstances, have a phase coherence in either the particle displacement or strains. It does however have a coherent acoustoelectric field which varies with the wavelength of the collective disturbance.

Other effects of the piezoelectric nonlinear interaction can be grouped under the heading of mixing interactions. These interactions redistribute the energy among different sound wave modes. If the mixing collisions thermalize the acoustic energy in a time of the order of τ , then any description of the acoustic system over times $t > \tau$ must be in terms of an acoustic fluid, as acoustic energy cannot be assigned to a specific sound wave over such times.

When discussing energy transport over times $t < \tau$, one can assign energy to specific modes and the solution of Sec. III is proper. In this case, the collective waves are convective or dynamic waves. The velocity is close to that of ordinary sound, only differing by the admixture of waves off the central axis which must travel at some angle to the collective propagation. This type of propagation is similar to the usual concept of phonon flux;

however, the acoustoelectric effects make the flux mutually attractive. The result of the attraction is to tend to maintain additional coherence in the flow.

For collective wavelengths and period long compared to free paths and mixing times, a second sound wave is the solution and this travels at a velocity slower than the velocity of sound by a factor $1/\sqrt{3}$. The mixing relaxation rate depends on the individual sound wave frequency, being fastest for higher frequencies. Since a wide range of frequencies from ultrasonic through hypersonic are piezoactive, sound waves capable of both types of propagation can be present simultaneously.

The solution for the mixing free path was based on a very simplified model, and the sound waves which can participate in a second sound wave are not clearly specified. In particular, the coherent gain present in these systems was ignored. Coherent gain lengthens the mixing free path and this effect is amplified by the reduction of the number of interacting modes which contribute to the damping of a given mode. To achieve a significant number of sound modes capable of propagating second sound requires a delicate balance of gain, which must be large enough to amplify the wave but small enough not to disrupt the mixing interaction rate. It appears that this balance can be attained because second sound waves have most probably been observed in CdS. 10

The role of electrons other than that of providing a source of gain and nonlinearity for the sound waves has been ignored in this paper. Certain complications in the propagation of collective waves arise when the electrons contribute dynamically. Such phenomena as plasma effects and drift velocity waves dominate the collective behavior. These effects are limited to lower frequency collective waves: viz, for CdS, $\Omega < 10^6$ sec⁻¹. A discussion and analysis of such waves has appeared in other publications. 3,11

REFERENCES

- 1. D. L. White, J. Appl. Phys. 33, 2547 (1962).
- 2. A. R. Hutson, Phys. Rev. Letters 9, 296 (1962).
- 3. H. R. Carleton, H. Kroger and E. W. Prohofsky, Proc. IEEE 53, 1452 (1965).
- 4. H. N. Spector, Phys. Rev. 127, 1084 (1962).
- 5. H. Kroger, Appl. Phys. Letters 4, 190 (1964).
- 6. B. Tell, Phys. Rev. 136, A772 (1964).
- 7. E. W. Prohofsky and J. A. Krumhansl, Phys. Rev. <u>134</u>, 1403 (1964); R. A. Guyer and J. A. Krumhansl, Phys. Rev. <u>134</u>, 1411 (1964); P. Kwok and P. Martin, to be published; E. W. Prohofsky, Phys. Rev. <u>134</u>, A1302 (1964); R. W. Damon, H. Kroger and E. W. Prohofsky, Proc. IEEE <u>52</u>, 912 (1964).
- 8. R. Nava, R. Azrt, I. Ciccarello and K. Dransfeld, Phys. Rev. 134, A581 (1964).
- 9. I. S. Ciccarello and K. Dransfeld, Phys. Rev. 134, A1517 (1964).
- 10. H. Kroger, E. W. Prohofsky and R. W. Damon, Phys. Rev. Letters 11, 246 (1963).
- 11. E. W. Prohofsky, Phys. Rev. <u>136</u>, A1731 (1964); H. Kroger, E. W. Prohofsky and H. R. Carleton, Phys. Rev. Letters <u>12</u>, 555 (1964).